

Physics Foundations



Section A: Dimensional Analysis

Most commonly seen SI units:

Length, Mass, Time ← Fundamental

(m) (kg) (s)

- useful to know common conversions between non SI units such as

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\vec{N} = N - m$$

and what are the SI units of stuff like energy

$$[E] = [mc^2] = M(LT^{-1})^2 = ML^2T^{-2}$$

- note: angles have no dimensions as well as the trigonometric functions and the exponential

How to use dimensional analysis

$$\text{constant} = \pi = Q_1^\alpha Q_2^\beta Q_3^\gamma \dots \rightarrow \text{Buckingham Pi theorem}$$

\downarrow dimensionless $Q_i = \text{variable} \neq i$

Section B: Thermodynamics

- energy exchange equilibrium between systems
 - ↳ definition of a system
 - ↳ definition of a state

↳ intensive / extensive variables

↳ open and closed systems

↳ different types of "walls" for closed systems

↳ definition of equilibrium

→ zeroth law of Thermodynamics

→ different processes:

- isothermal
- isobaric
- isochoric
- adiabatic

• definition of temperature (in Kelvins)

• linear thermal expansion

$$L = L_0 + \Delta L =$$

linear expansion coefficient $\alpha = \frac{1}{L_0} \frac{\Delta L}{\Delta T}$

• volume expansion: (similar to linear)

$$V = V_0 + \Delta V$$

volume expansion coefficient $\beta = \frac{1}{V_0} \frac{\Delta V}{\Delta T}$

moles $B = 3\alpha$ if isotropy (same in every)

$Q = n C \Delta T$ (specific heat capacity every direction)
 heat \downarrow number of moles \downarrow

two variants, one for moles & one for mass so look out for C

$Q = m c \Delta T$
 mass \downarrow

latent heat

$Q = m L \rightarrow$ latent heat
 heat energy \downarrow mass

Heat flow:

conduction $\frac{dQ}{dt} = -KA \frac{dT}{dx}$

check notes for diagram Q



2. convection

3. Radiation $P = \frac{dQ}{dt} = \epsilon \sigma A T^4$

Ideal gas law:

$pV = nRT$

p - pressure

V - volume

n = number of moles

$R = 8.315 \text{ J mol}^{-1} \text{ K}^{-1}$

or

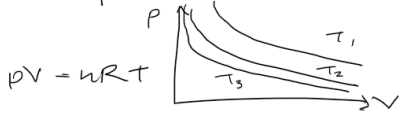
$pV = N k_B T$

T - temperature

$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$

N = number of atoms

pV diagrams



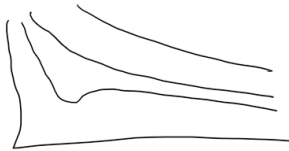
$p \propto \frac{1}{V}$ $T_1 > T_2 > T_3$

assumptions made:

1. Molecules finite volume
2. Intermolecular forces

$(p + \frac{a n^2}{V^2})(V - nb) = nRT$

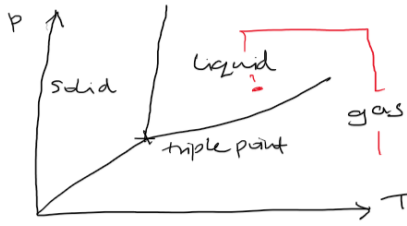
vander waals equation of state



analysis of this graph in your notes (week 3)

Phase diagrams

= analysis of it in notes (week 3)



• Kinetic-molecular model of an ideal gas

derivation in notes $\Rightarrow pV = \frac{N}{3} m \langle v^2 \rangle = Nk_B T$

\downarrow mass of a molecule \downarrow average of the square of the velocity

\Rightarrow average kinetic energy $= \frac{3}{2} k_B T$

$k_B T \sim$ energy scaling

average time between collisions

$$t_{mean} = \frac{v}{4\pi r^2 v_{rms} N}$$

r is the radius of the molecule

$$v_{rms} = \sqrt{\langle v^2 \rangle}$$

mean free path - average distance travelled between collisions

heat capacity at constant volume

ideal gas $C_V = \frac{3R}{2}$

$\lambda = v_{rms} t_{mean}$

• use $pV = nRT$ to deduce these values

Equipartition of energy

\rightarrow each degree of freedom contributes $\frac{1}{2} k_B T$ of energy per atom

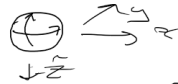
translation in 3D: $\frac{3}{2} k_B T$

rotation: $k_B T$

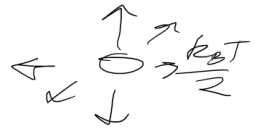
vibration: $k_B T$

if all apply, we get $\frac{f}{2} k_B T$

along θ & ϕ spherical polar directions



and meanings of Q, W positive or negative



1st law of thermodynamics

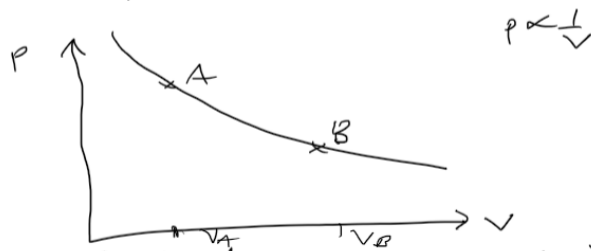
$$\Delta U = Q - W$$

for a given pV diagram

know how to use $W = \int_{V_1}^{V_2} p dV$

Isothermal expansion of ideal gas

Isothermal expansion of ideal gas



$$W = nRT \ln \left(\frac{V_B}{V_A} \right)$$

$$Q = W$$

+ analysis of pV diagrams (week 4)

⇒ for a loop back to the same point
 $\Delta U = 0$ but $Q, W \neq 0$

$$\text{in general } dU = dQ - dW$$

$$C_p = C_v + R = \frac{\gamma R}{2}$$

↳ specific heat capacity at constant pressure for an ideal gas

$$\gamma = \frac{C_p}{C_v}$$

→ Adiabatic expansion of an ideal gas $Q = 0$

+ for isotherms: $p_1 V_1 = p_2 V_2$

for adiabatic process: $pV^\gamma = \text{constant}$

Summary of processes:

a) Isothermal $\Delta T = 0$ $\Delta U = 0$ $pV = \text{constant}$

$$W = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \left(\frac{V_2}{V_1} \right)$$

b) Adiabatic: $Q = 0$
 $pV^\gamma = \text{constant}$
 $TV^{\gamma-1} = \text{constant}$

$$W = -\frac{1}{\gamma-1} (p_2 V_2 - p_1 V_1)$$

c) Isochoric: $\Delta V = 0$ then $W = 0$
 $Q = \Delta U = n C_v \Delta T$

$$p \downarrow \quad \Delta T < 0 \quad Q < 0$$

$$p \uparrow \quad \Delta T > 0 \quad Q > 0$$

d) Isobaric $\Delta p = 0$ $\therefore W = \int p dV = p \Delta V$
 $\Delta U = Q - \int p dV = Q - p \Delta V$
 $Q = n C_p \Delta T$

- second law of thermodynamics
- heat engine in notes } pV diagrams
- fridge and heat pump. }
- Otto cycle • Diesel cycle • Carnot cycle

important one
know the processes
involved
+ efficiency
calculations

- reversible & irreversible processes
- entropy definition → important
↳ measure of a system's randomness

$$\Delta S \geq 0 \quad \text{i.e. entropy can never decrease for irreversible processes}$$

for a system where we add heat Q
(consider ideal gas)

$$dQ = \frac{nRT}{V} dV = \frac{1}{Rn} \frac{dQ}{T}$$

$$\text{where } dS = \frac{dQ}{T}$$

∴ change in entropy ΔS given by:
 $\Delta S = \frac{Q}{T} \rightarrow$ heat input

for a non-isothermal process:

$$\Delta S = \int_{\text{state 1}}^{\text{state 2}} \frac{dQ}{T}$$

$$1^{\text{st}} \text{ law of thermo} \equiv dU = TdS - pdV$$

2nd law = "when all systems involved in a process are included, entropy remains the same or increases."



Section B: Thermodynamics is long but know when to use certain PV diagrams and you will be all good :)

Section C: Waves

2 types of wave waves:

- Mechanical waves
- Matter waves
- Electromagnetic (EM) waves

waveform: $u(x,t) = f(x \pm ct)$
↳ speed of wave (speed of light for EM waves)

"-" ⇒ along \hat{c} direction

"+" ⇒ along $-\hat{c}$ direction

$$k = \frac{2\pi}{\lambda}$$

typically:

$$u(x,t) = A \cos(kx - \omega t)$$

A = amplitude

k = wavenumber

ω = angular frequency

$$\omega = 2\pi f = 2\pi \nu$$

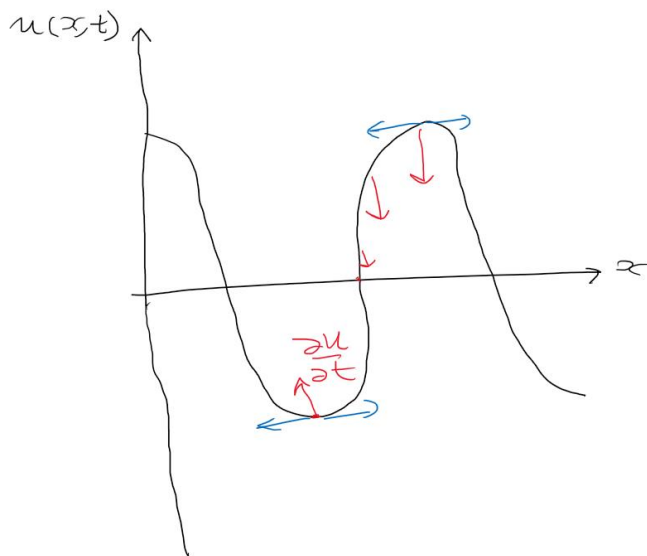
$$c = \frac{v_0}{R} = f\lambda$$

what is a wave peak?
 " " " wavelength
 " " " period

- 1) transverse: the medium the wave propagates through vibrates perpendicularly to the direction of travel of the wave
- 2) longitudinal: same as above except parallel instead of perpendicular.

$$\frac{\partial u}{\partial t} = \text{speed at which vibrating atom on the wave is travelling}$$

≠
speed of the wave



red \Rightarrow transverse
 blue \Rightarrow longitudinal

recall that $e^{i\theta} = \cos\theta + i\sin\theta$

$$\Rightarrow u(x,t) = \text{Re}[A \exp(i(kx - \omega t))]$$

or for notational reasons we drop the $\text{Re}(\cdot)$ part

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{wave equation (one-dimensional)}$$

where we note:

$$c = \frac{\omega}{k}$$

for EM waves:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{hence } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}$$

where E is the electric field

Principle of superposition:

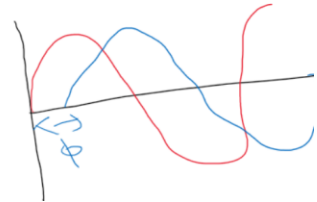
→ add waves together → find total wave

phase:

$$u(x,t) = A \cos(kx - \omega t + \phi)$$

↳ phase of the wave

→ phase leads to interference effects (constructive & destructive)



waves of same frequency & wavelength (assuming equal amplitudes)

$$u_1(x,t) = A \cos(kx - \omega t)$$

$$u_2(x,t) = A \cos(kx - \omega t + \phi)$$

$$u_1 + u_2 = 2u_1 = 2u_2 \quad \text{if } \phi = 2n\pi \quad \forall n \in \mathbb{Z} \quad \text{constructive}$$

$$u_1 + u_2 = 0 \quad \text{if } \phi = (2n+1)\pi \quad \forall n \in \mathbb{Z} \quad \text{destructive}$$

ϕ any value however

for the superposition of waves:

$$\text{group velocity} = \frac{d\omega}{dk} = v_g \quad \&$$

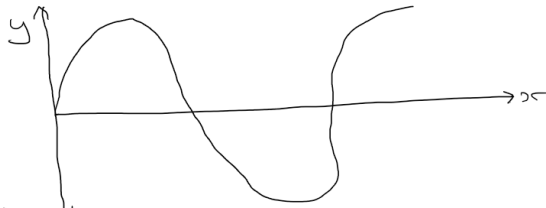
$$\text{phase velocity} = \frac{\omega}{k} = v$$

waves are non-dispersive if $v_g = \frac{d\omega}{dk} = \frac{\omega}{k} = v$

initially, waves are dispersive if $v_g \neq v$

$\omega(k)$ dispersion relations

example of a mechanical wave: waves on a taut string



deduced in notes

that $\mu \frac{d^2 y}{dt^2} = T \frac{d^2 y}{dx^2}$

shares similarities to the wave equation
 $T = \text{Tension}$, $\mu = \text{mass per unit length}$

$$v = \sqrt{\frac{T}{\mu}}$$

speed of the wave

waves can transfer energy

power $P = -T \frac{\partial y}{\partial x} \frac{\partial y}{\partial t}$

if we have

$$y(x,t) = A \cos(kx - \omega t)$$

$$\Rightarrow P = TA^2 k \omega \sin^2(kx - \omega t)$$

$$v = \frac{\omega}{k}, v^2 = \frac{T}{\mu} \Rightarrow k = \omega \sqrt{\frac{\mu}{T}}$$

$$P(x,t) = \omega^3 A^2 \underbrace{\sqrt{\mu T}}_Z \sin^2(kx - \omega t)$$

$Z = \text{impedance}$

$$\langle P(x,t) \rangle = \frac{1}{2} \omega^3 A^2 Z$$

where note:

$$\langle P(x,t) \rangle \propto A^2 \omega^3$$

true for mechanical waves

for longitudinal waves along a rod (one dimension)

strain: $\frac{\Delta \sigma}{\Delta x} = \frac{\Delta l}{l}$ (from A-level)

stress: $\frac{F}{A} = Y \frac{\partial \sigma}{\partial x}$ as $\Delta x \rightarrow 0$

Young's modulus

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\rho}{Y} \frac{\partial^2 \sigma}{\partial t^2}$$

$\rho = \text{density}$

& so $v = \sqrt{\frac{Y}{\rho}}$



for a shear wave we use G (shear modulus) instead of γ

$$v = \sqrt{\frac{G}{\rho}}$$

waves in a gas: using the general waveform:

$$u(x,t) = A \cos(kx - \omega t)$$

$$\Rightarrow P = AkB \sin(kx - \omega t)$$

where B is the Bulk modulus

$$\& \quad v_{\text{sound}} = \sqrt{\frac{B}{\rho}}$$

speed of sound in an ideal gas:

for sound waves, in this system it is considered an adiabatic process

from thermal: $pV^\gamma = \text{constant}$

$$\gamma = \frac{C_p}{C_v}$$

$$B = -V \frac{\partial p}{\partial V}$$

\Rightarrow

$$B = \gamma p \quad \& \text{ using } v_{\text{sound}} = \sqrt{\frac{B}{\rho}} \quad \& \quad \rho = \frac{M}{V} = \frac{Nm}{V}$$

$$\Rightarrow v_{\text{sound}} = \sqrt{\frac{\gamma p V}{Nm}}$$

& using $pV = Nk_B T$

$$v_{\text{sound}} = \sqrt{\frac{\gamma k_B T}{m}}$$

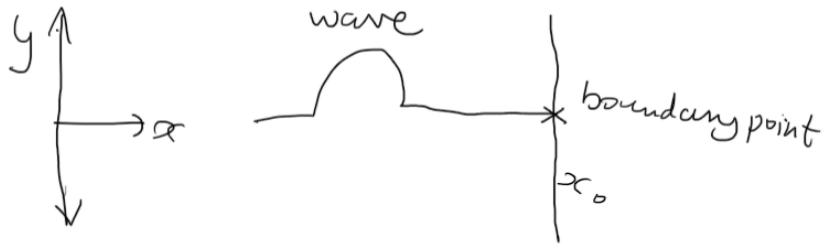
from thermal we said

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

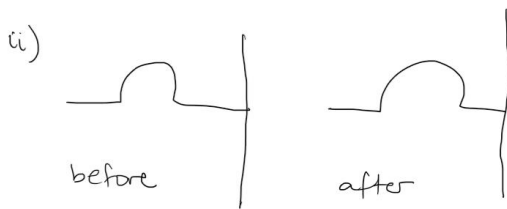
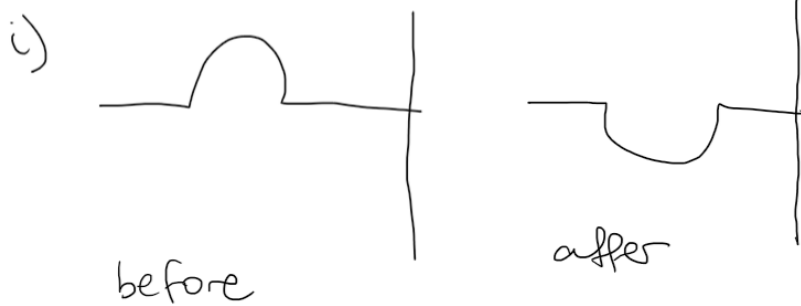
$$v_{\text{sound}} = v_{\text{rms}} \cdot \left(\frac{\gamma}{3}\right)^{\frac{1}{2}}$$

what happens to waves at boundaries, consider two cases:

- i) fixed ends
- ii) free ends



- i) $y(x=x_0) = 0$ comes back inverted
 ii) $\frac{\partial y}{\partial x} \Big|_{x=x_0} = 0$ comes back not inverted



- case where two strings are joined together
 → when $\mu_1 < \mu_2$, $\mu_1 = \mu_2$ & $\mu_1 > \mu_2$

where $Z = \sqrt{\mu T}$ → tension

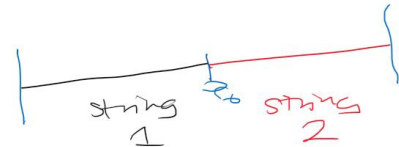
boundary condition:

$$y_1(x=x_0) = y_2(x=x_0)$$

if y_i = incident wave, y_r = reflected wave &
 y_t = transmitted wave

$$y_i + y_r = y_t$$

& taking $y(x,t) = A \sin(kx - \omega t)$



$$\Rightarrow A_i + A_r = A_t$$

+ tension at the join is the same

$$\Rightarrow \frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x} \quad \text{at } x=x_0$$

lets assume same T, ω , different $k(\lambda)$

$$v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \quad \text{where } Z = \sqrt{\mu T}$$

$$v \propto Z$$

$$y_i(x,t) = A_i \sin(k_i x - \omega t)$$

right
travelling

$$y_r(x,t) = A_r \sin(k_r x + \omega t)$$

left
travelling

$$y_t(x,t) = A_t \sin(k_t x - \omega t)$$

$$\Rightarrow \text{transmission coefficient } T = \frac{A_t}{A_i}$$

$$\text{reflection coefficient } R = \frac{A_r}{A_i}$$

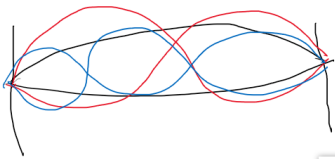
$$\text{for the power of the waves: } P_i = P_t + P_r$$

$$\begin{aligned} \text{standing wave: } u_{\text{total}}(x,t) &= A \cos(kx - \omega t) - A \cos(kx + \omega t) \\ &= 2A \sin(kx) \sin(\omega t) \end{aligned}$$

\neq travelling wave

observe what happens when $kx = n\pi \quad n \in \mathbb{Z}$

$$\begin{aligned} xk &= n\pi + \frac{\pi}{2} \\ &= (n + \frac{1}{2})\pi \quad n \in \mathbb{Z} \end{aligned}$$



standing waves & modes:

$$\lambda = \frac{2L}{n} \quad n \in \mathbb{N}$$

$$k = \frac{n\pi}{L}$$

$$f = \frac{v}{\lambda}$$

$$\omega = \frac{n\pi v}{L}$$

• waves in an open ended pipe (in notes)

The Doppler Effect

→ the apparent change in frequency when a source of waves is moving towards or away from a receiver
 ⇒ occurs whether the source or the receiver is moving

e.g red & blueshift of stars
 must treat sound & light differently

• if source & receiver moving towards each other frequency increases & vice versa

for sound: $f' = f \left(\frac{c + u_o}{c + u_s} \right)$

\downarrow frequency measured by observer \downarrow frequency emitted

$u_o =$ observer speed
 $u_s =$ source speed

$u_o > u_s$

~~~~~

for EM waves:  $f' = f \sqrt{\frac{c-v}{c+v}}$

the nature of light:

• light an EM wave ⇒ propagate through a vacuum  
 Maxwell obtained:

$$\frac{\partial^2 E}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \quad \epsilon_0 = \text{permittivity of free space}$$

$$\frac{\partial^2 B}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2} \quad \mu_0 = \text{permeability of free space}$$

→ wave equation form

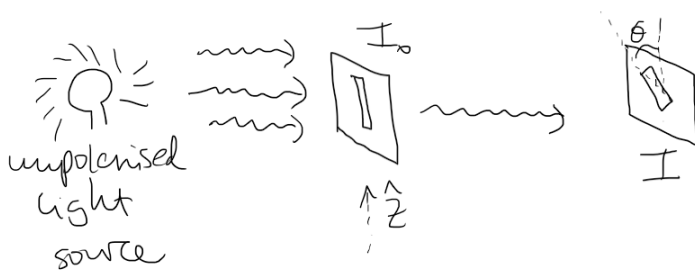
$$\Rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

→ oscillating E & B fields      → transverse

$$\vec{E}(r,t) = \vec{j} E_0 \cos(kr - \omega t)$$

$$\vec{B}(r,t) = \vec{k} B_0 \cos(kr - \omega t)$$

Polarisation: electric field only oscillating in one direction when passed through a polarising filter  
 → only component of E-field parallel to polarisation axis passed through

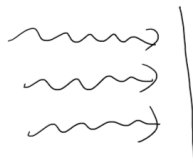


Malus' Law  
 $I = I_0 \cos^2(\theta)$

for EM waves  $I = \frac{1}{2} \epsilon_0 c E_0^2$

intensity independent of frequency unlike mechanical waves

pressure from incident light

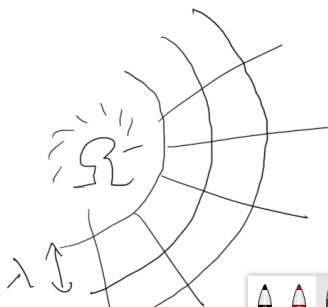


pressure  
 $\frac{dp}{dt} = \frac{I}{c}$



Propagation of light:

wavefronts represent points of a wave that are in phase

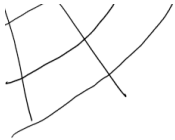


wavefronts  $\perp$  direction of light rays

Huygens' principle:

new wavefront is the sum of the wavelets from

previous



new wavefront is the sum of the wavelets from the point sources on the previous wavefront



diagram in notes

- simplistic model to account for propagation
- does not account for interference
- we would expect backward travelling wave but this does not account for it.

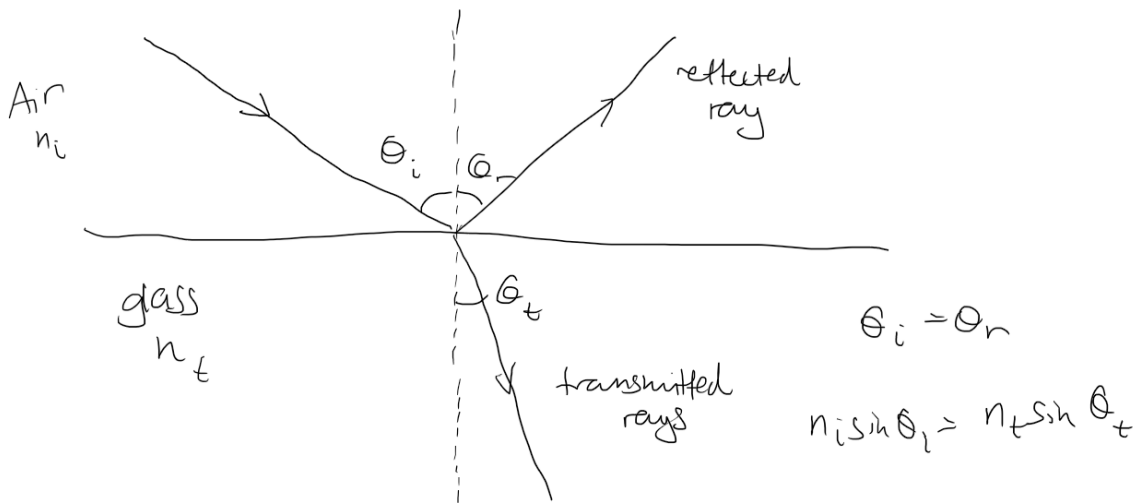
Light in matter: the refractive index

light travels through matter via a continual process of scattering by the electrons in the material

$$n = \frac{c}{v}$$

$n \equiv$  refractive index  
 $v =$  velocity of the light in the material  
 $c =$  speed of light in a vacuum

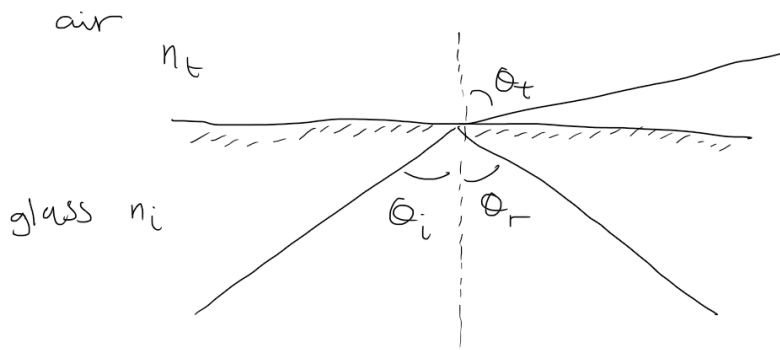
Snell's law : waves at a boundary



transmitted coefficient :  $T = \frac{4n_i n_t}{(n_i + n_t)^2}$

reflection coefficient :  $R = \frac{(n_i - n_t)^2}{(n_i + n_t)^2}$

Total internal reflection: when  $n_i > n_t$



$$n_i \sin \theta_i = n_t \sin \theta_t$$

$$\sin \theta_t = \frac{n_i}{n_t} \sin \theta_i$$

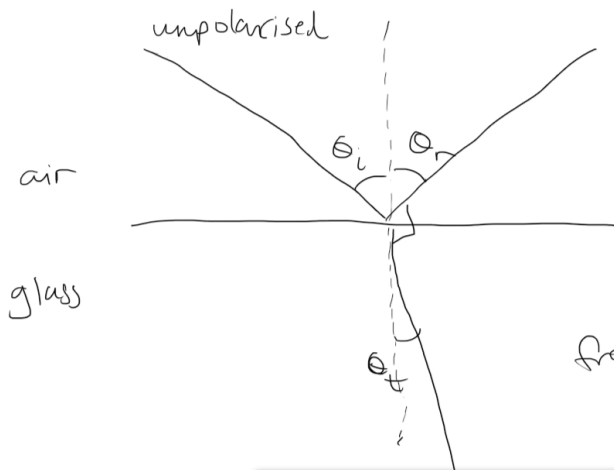
i.e.  $\theta_t > \theta_i$

when  $\theta_t$  reaches  $90^\circ$

$\theta_c \equiv$  critical angle

$\theta_i = \theta_c$  & any  $\theta_i > \theta_c$  there is no transmitted ray

Brewster's law:  
- light can be polarised by reflection



angle between reflected & transmitted ray is  $90^\circ$

from Snell's law, we know that

$$\tan \theta_i = \frac{n_t}{n_i}$$

$\theta_i \rightarrow \theta_p$   
↳ polarisation angle.

## Interference of two waves

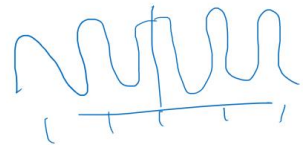
- need coherent waves  
→ same phase & constant over time
- monochromatic (same  $\lambda$ )

see week 10 notes for set up & details

constructive interference :  $n\lambda = d \sin \theta$   $d =$  source separation  
 $n = 0, 1, 2, \dots$

destructive interference :  $(n + \frac{1}{2})\lambda = d \sin \theta$

two point sources of light:



$$E_1 = E_0 \cos(kx_1 - \omega t); \quad E_2 = E_0 \cos(kx_2 - \omega t)$$
$$E_{\text{total}} = E_1 + E_2 = 2E_0 \cos\left(k\left(\frac{x_1+x_2}{2}\right) - \omega t\right) \cos\left(\frac{1}{2}(x_1 - x_2)\right)$$

$$\Delta x = x_1 - x_2$$

constructive if  
equals  $\pm 1$

$$\Rightarrow n\lambda = \Delta x$$

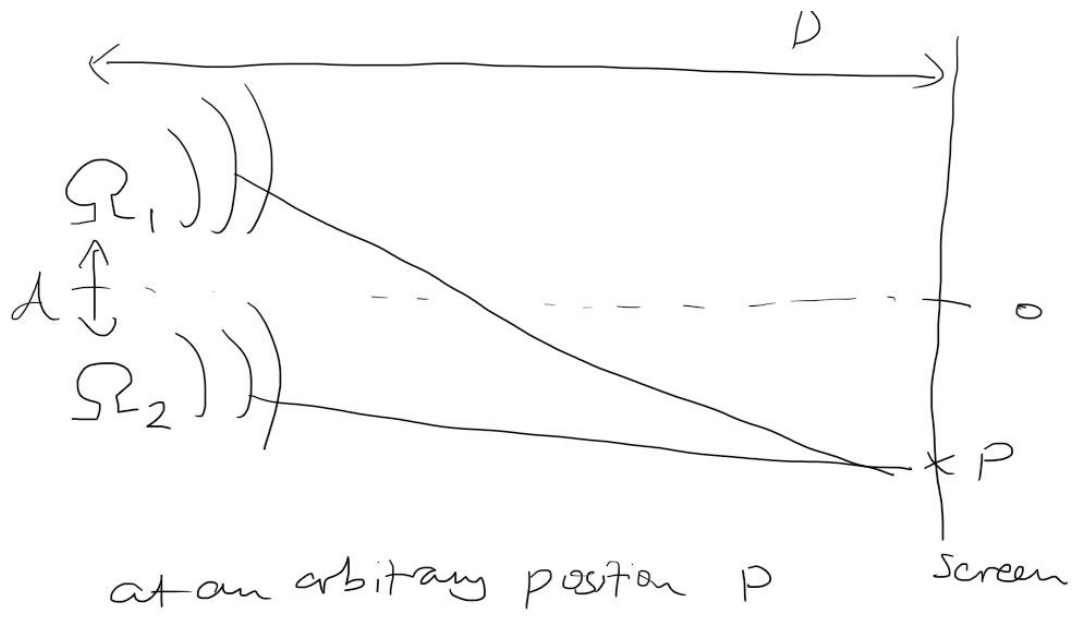
$\Rightarrow$  amplitude of  $2E_0$

defined  $I = \frac{1}{2} \epsilon_0 c E_0^2$  before of a single wave

$$\therefore I_{\text{total}} = \frac{1}{2} \epsilon_0 c (2E_0)^2 = 2\epsilon_0 c E_0^2 = 4I_1 = 4I_2$$

$I_1, I_2$  are the intensities of the separate waves.

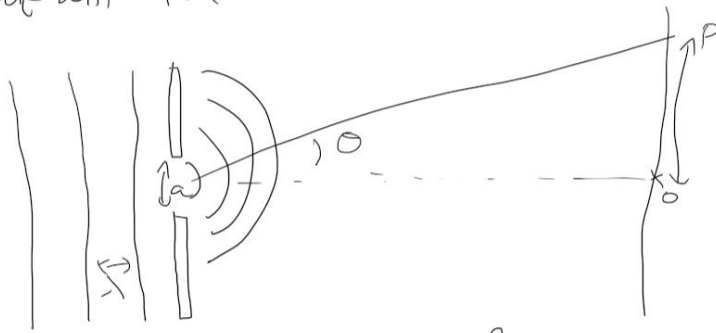




$$I = I_0 \cos^2\left(\frac{k \Delta r}{2}\right)$$

$$\text{for } D \gg d \quad I = I_0 \cos^2\left(\frac{k d \sin \theta}{2}\right)$$

single slit diffractor

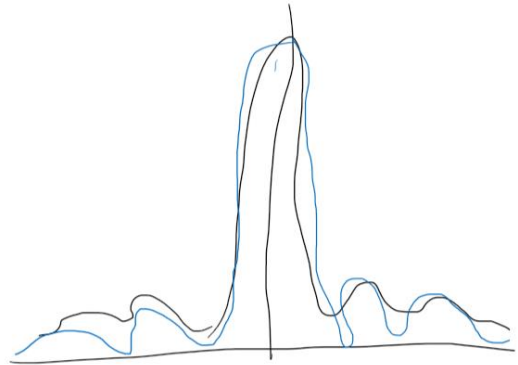


$$I(\theta) = I_0 \left( \frac{\sin\left(\frac{k a \sin\theta}{2}\right)}{\frac{k a \sin\theta}{2}} \right)^2$$

for minima:

$$\frac{k a \sin\theta}{2} = n\pi$$

for maxima  $\tan\beta = \beta$   
where  $\beta = \frac{k a \sin\theta}{2}$



Double slit: similar to single slit

$$I(\theta) = I_0 \cos^2\left(\frac{k d \sin\theta}{2}\right)$$

